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Study of Parameter Sensitivity of River Bedform Model Due to Delta Formation Phenomena

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Abstract

The phenomenon of delta formation in estuarial region is described using the mathematical model of sediment transport on some interval of river region. The model is stated in second order ordinary differential equation of the river depth. As a consequence, the sediment transport could be restricted in a half of river width. The sensitivity of parameter is shown by consider the river bed formation and the level sets of the river depth and its derivative.

Keywords: River bed, sediment transport, river depth, river width, parameter sensitivity.

1. Introduction

Being interested in the evolution of a river, especially the shape of its bedform developed in time due to the sediment transport, will make us to think about the existence of steady river bed. The discussion of river bedform in steady state condition as a solution of mathematical model of sediment transport is found on [Ratianingsih, 1996]. The appearance parameter b on the model is described the nonlinearity order of sediment transport concentration and the velocity of river flow. The discussion of the parameter contribution was considered on [Ratianingsih, 1998] which gives some river bed formations related to the parameter varying.

The effect of the parameter varying will be analyzed numerically by consider the level sets of the river depth and its derivative. The changing of river depth and its derivative will cause the changing of river bedform that tend to delta formation phenomena in estuarial region.

2. The Problem

Delta formation phenomena that are caused by the deposition of sediment in river bed usually found in estuarial region. The deposition of sediment will change the river depth that makes the sediment transport in transversal and lateral (main stream) direction will not balance anymore.

This paper will investigate the contribution of the sensitivity of parameter b to the formation of delta by consider the behavior of the solution of model. The solution, described by the phase portrait of the river depth and its derivative, is numerically observed respect to the behavior of river bed changing. The river bed changing is shown by the changing of level sets when the parameter b is varied.

3. The Mathematical Model

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The mathematical model of horizontal sediment distribution in such cross-section of river could be found on [Ratianingsih, 1996]. The model will derive the differential equation of the river depth that will be redesign in form of initial value problem of second order ordinary differential equations as follow

$$h'' = f(h),\ h(0) = h_o,\ h'(0) = 0 \tag{1}$$

where $f(h) = -\dfrac{BA}{h^{-(1+2B)}}$, $A > 0$ is derived from steady state model of sediment transport in such river cross-section and *h(y)* is the river depth on such position of cross-section *y*.

The form of equation (1) is design in order to face the singularity problem that found on [Ratianingsih, 1996]. Consequently, we get an autonomous differential equation that guarantees the symmetry solution.

4. The Sensitivity of Parameter

We could stated the equation (1) into system of differential equation

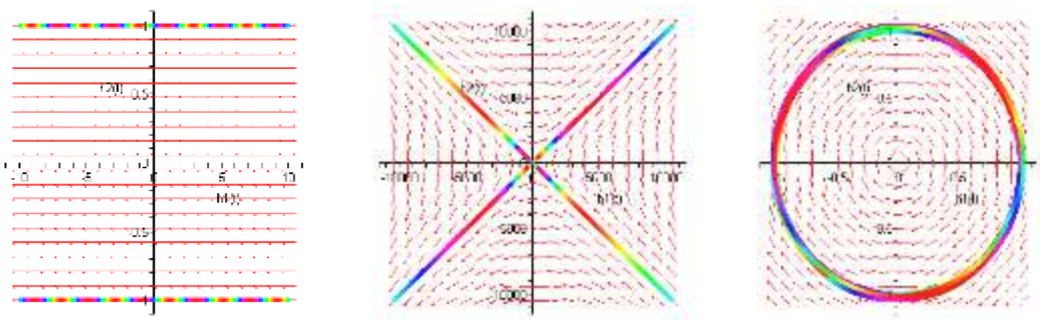
$$\begin{pmatrix} h_1{}' \\ h_2{}' \end{pmatrix} = \begin{pmatrix} h_2 \\ \lambda h^{-(1+2b)} \end{pmatrix} \tag{2}$$

where $\lambda = -BA$. We could construct such function $H(h_1, h_2)$ as follow

$$H = \frac{\lambda}{2B} h_1{}^{-2b} + \frac{1}{2} h_2{}^2 \tag{3}$$

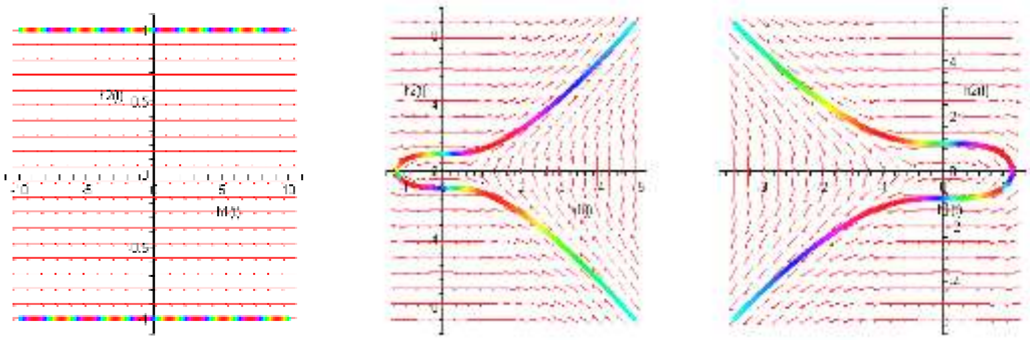
where the derivative of H must be zero. From equation (3), we could found level sets of $\{(h_1, h_2) | H(h_1, h_2) = C\}$ for such constant of C.

The sensitivity of parameter b will be analyzed by consider the phase portrait of equation (2) for some value of λ when the parameter is varied. The phase portrait is determined numerically by the level set. The result, for some value of *b*, is figured in figure (1) until figure (8).



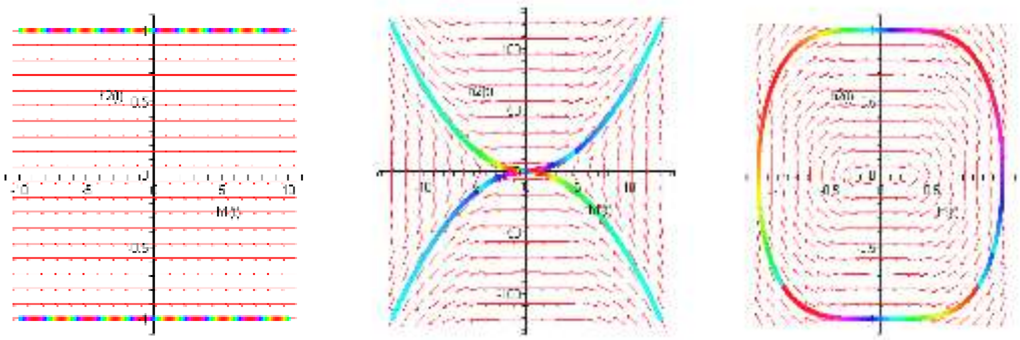
(a) (b) (c)

Figure 1. The phase portrait of (2) for –(1+2b) = 1, respectively for (a) $\lambda = 0$, (b) $\lambda > 0$ and (c) $\lambda < 0$



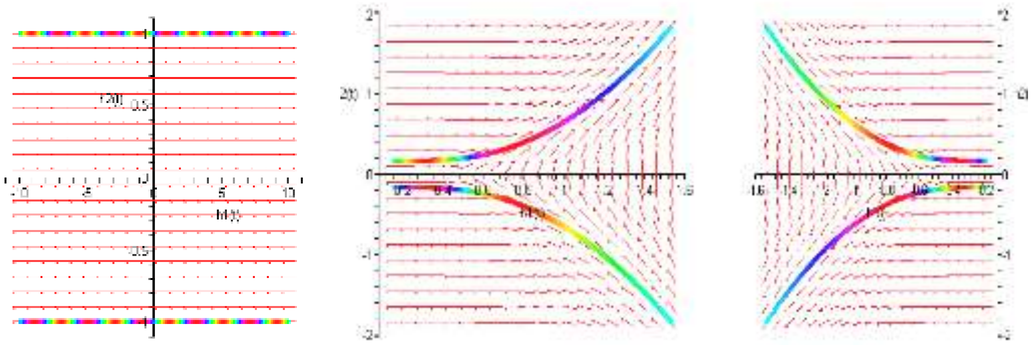
(a) (b) (c)

Figure 2. The phase portrait of (2) for $-(1+2b) = 2$, respectively for (a) $\lambda = 0$, (b) $\lambda > 0$ and (c) $\lambda < 0$



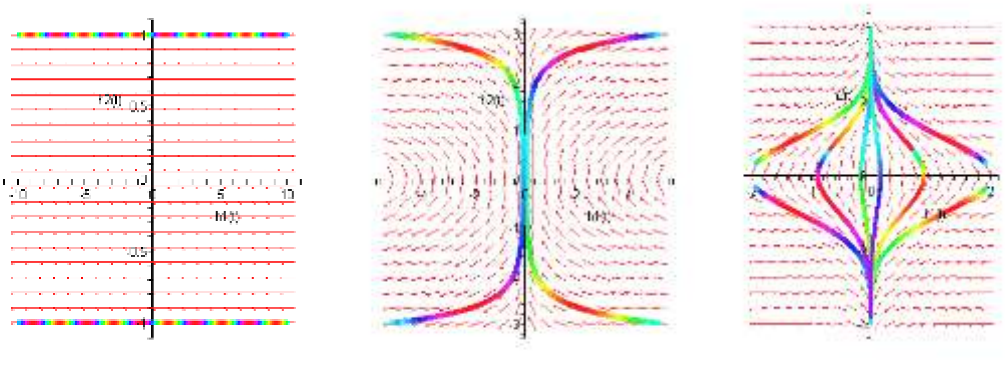
(a) (b) (c)

Figure 3. The phase portrait of (2) for $-(1+2b) = 3$, respectively for (a) $\lambda = 0$, (b) $\lambda > 0$ and (c) $\lambda < 0$



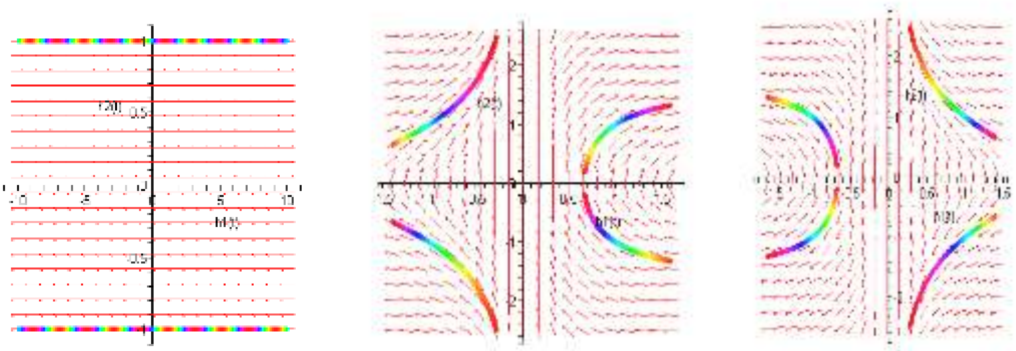
(a) (b) (c)

Figure 4. The phase portrait of (2) for $-(1+2b) = 4$, respectively for (a) $\lambda = 0$, (b) $\lambda > 0$ and (c) $\lambda < 0$



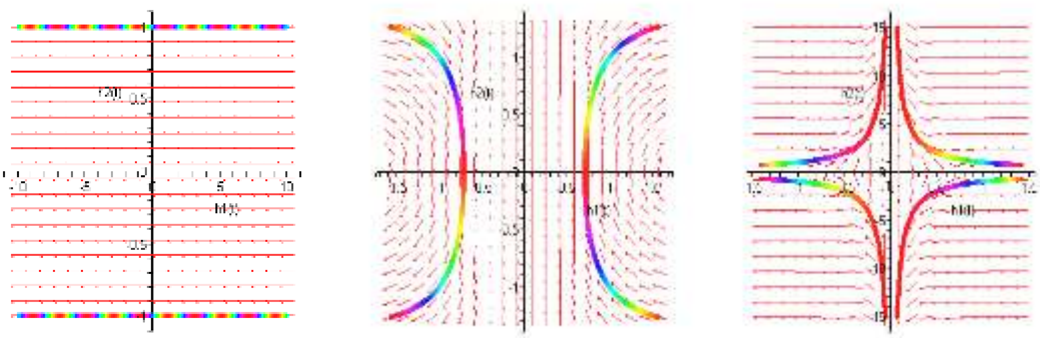
(a) (b) (c)

Figure 5. The phase portrait of (2) for $-(1+2b) = -1$, respectively for (a) $\lambda = 0$, (b) $\lambda > 0$ and (c) $\lambda < 0$



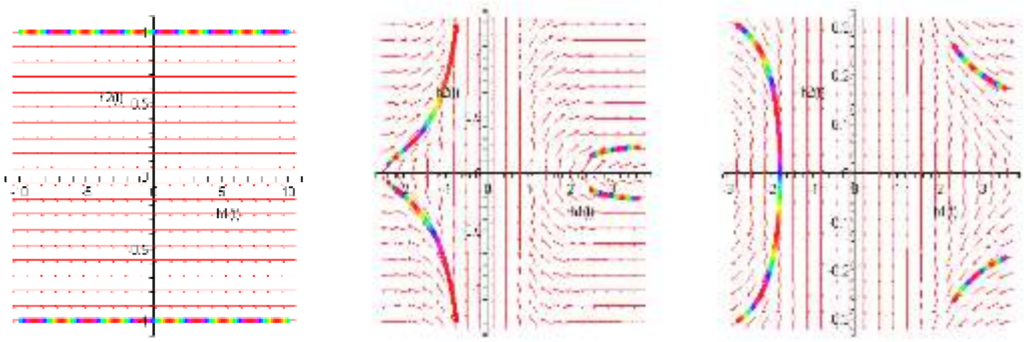
(a) (b) (c)

Figure 6. The phase portrait of (2) for $-(1+2b) = -2$, respectively for (a) $\lambda = 0$, (b) $\lambda > 0$ and (c) $\lambda < 0$



(a) (b) (c)

Figure 7. The phase portrait of (2) for $-(1+2b) = -3$, respectively for (a) $\lambda = 0$, (b) $\lambda > 0$ and (c) $\lambda < 0$



(a) (b) (c)

Figure 8. The phase portrait of (2) for –(1+2*b*) = -4, respectively for (a) $\lambda = 0$, (b) $\lambda > 0$ and (c) $\lambda < 0$

5. Conslusion and Open Problem

From the result that presented in section 4 we conclude that the variation of the value of parameter *b* will gives the variation of solution which is determined from the level sets of *H*. We also found that we have a constant solution which determines the maximum depth of river. The zero solution is unstable almost everywhere, especially the case of $b = -1$ and $b = -2$ that give a periodic solution for negative value of λ. It means that the steady river bed founded on [Ratianingsih, 1996] is sensitively changing due to the varied value of parameter *b*. In other word the steady river bed tends to move because of the sensitivity of the parameter and the changing of level sets refer to the changing of river bedform. The changing of river bedform could be interpreted as a deposition of sediment at the river bed that lead to the formation of delta in estuarial region.

The discussion of the river bed motion could be continued by consider the impact of nonlinear term in order to get non linear river bed profile.

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